Multifunctionality of Cellular Metal Systems

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Abstract

Cellular metals have ranges of thermomechanical properties that suggest their implementation in ultralight structures, as well as for impact/blast amelioration systems and for heat dissipation media. The realization of these applications requires that their thermostructural benefits over competing concepts be firmly established. This overview examines the mechanical and thermal properties of this material class, relative to other cellular and dense materials. It also provides design analyses for prototypical systems.

1 Introduction

Cellular metals have combinations of mechanical, thermal and acoustic properties that provide opportunities for diverse thermostructural implementations [1-9]. The technologies include ultralight structures [10,11], impact absorbers [2,12], heat dissipation media and compact heat exchangers [13,14]. Successful implementation relies not just on their thermomechanical properties, but on additional attributes: low manufacturing cost, environmental durability and fire retardancy [15]. Because of this diversity, multifunctional representation/analysis is an essential element in the engineering strategy [16]. This article addresses two aspects of multifunctionality: one concerned with ultralight structures and the other with heat dissipation.

The benchmarks for comparison with sandwich skin construction comprise [10,11,24,27]: (i) stringer-stiffened panels or shells, (ii) honeycomb panels and (iii) hollow tubes. Through decades of development, all three have been optimized and provide performance targets that are difficult to supercede. Often the benefits of cellular metal system derive from an acceptable structural performance combined with lower costs or greater durability than competing concepts. For example, honeycomb panels comprising polymer composite face sheets with Al honeycomb core are particularly weight efficient: they can never be superceded by cellular metal construction strictly on a performance basis. However, such honeycombs have durability problems associated largely with water intrusion and they are relatively expensive [28]. They are also highly anisotropic and costly to configure as cores for curved structures. Cellular metal construction can become competitive on a performance/durability/affordability basis, particularly for shell structures and geometrically complex panels. Elaboration on these and other considerations comprises one theme of this article.

Open cell metals constitute a second opportunity. These materials have thermal attributes that enable applications as heat dissipation media and for thermal recuperation [13,14]. The attributes include the high thermal conductivity of the material comprising the borders, in combination with high internal surface area and propitious fluid transport dynamics. These enable high heat transfer rates that can be used effectively for either cooling or efficient heat exchange.

2 Scaling relations

2.1 Stiffness

Closed cell structures establish upper limits on stiffness. At the low relative densities of present interest, the Young's modulus, E, of such structures scales as [1,19,20,23,29]:

$$E/E_s = \alpha_1 \rho \tag{1}$$

where E_s is the modulus of the solid material comprising the cell walls. The coefficient α_1 depends on the geometric arrangement of cells. Honeycombs are anisotropic, with $\alpha_1 \approx 1$ for longitudinal loading. In the transverse direction, the stiffness is considerably lower. For tetrakaidecahedra, $\alpha_1 = 0.35$, with a weak dependence on the distribution of material between the borders and the walls [19,20]. For thin-walled spheres, high stiffness can also be obtained ($\alpha_1 = 0.35$), but only with f.c.c. packing and when the contact radius is relatively large. Open cell solids, unless specially configured as lattice structures, are susceptible to bending, causing their stiffness to be relatively low and subject to the scaling [1],

$$E/E_s = \alpha_2 \rho^2 \tag{2}$$

where α_2 is about unity.

Commercially available materials have stiffness lower than equation (1) [16,19-22]. The knockdown factors on α_1 are found to range from 2 to 50 (fig. 1). This knockdown effect arises because of morphological defects that induce bending and buckling deformations. The nature of these defects is elaborated in section 3. Moreover, the totality of available data for closed cell Al foams (fig.1) is more comprehensively represented by (2), rather than (1), with α_2 ranging from ~3 for the higher quality, low density material to ~1/2 for inferior materials. This phenomenological scaling has utility in the analysis of minimum weight structures.

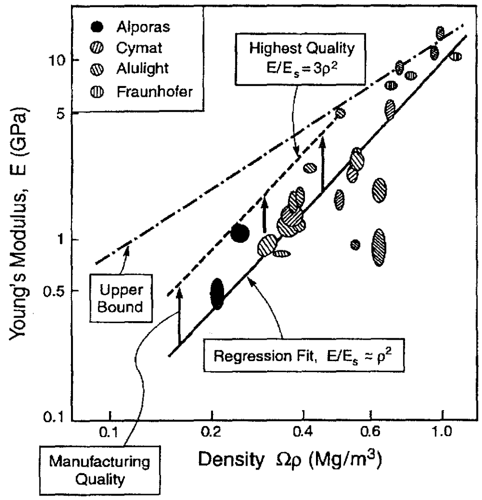

Fig. 1 Stiffness data for Al alloys.

2.2 Plastic Flow

The inelastic properties of cellular metals have not been as extensively studied as their stiffness. Accordingly, the scaling relations remain to be substantiated. Numerical simulations for various cell configurations indicate a negligibly small elastic region, because of localized yielding, followed by rapid strain hardening (even when the base material is perfectly plastic) and then a stress maximum σ_o. The available theoretical results for closed cell systems suggest a linear dependence on the density [1,20,23,29]:

$$\sigma_o / \sigma_s = \alpha_3 \rho, \tag{3}$$

where σ_s is the yield strength of the material comprising the cell borders. Results for the periodic tetrakaidecahedron, indicate that $\alpha_3 \approx 0.3$. But now, α_3 is significantly reduced upon distributing more of the material from the walls within the borders [19]. Results for bonded spheres with simple cubic packing [23] reveal that α_3 approaches 0.3 provided that the contact radius is relatively large. The yielding of open cell materials is limited by the bending stresses induced at the nodes, leading to the scaling [1]:

$$\sigma_o / \sigma_s = \alpha_4 \rho^{3/2} \tag{4}$$

where the coefficient, $\alpha_4 \approx 0.3$. Based on estimates of σ_s made using microhardness data, the knockdown factor on α_3 for commercial closed cell Al alloys is found to range from 4 to 100 [16,20,22,31]. The morphological defects discussed in section 3 are responsible. A phenomenological representation based on (4) may again apply and have utility for sandwich panel analysis. Commercial open cell materials seemingly satisfy (4) with essentially no knockdown on α_4 [16,19].

3 Morphological defects

3.1 Morphological Rules

The degrading effects of large bending moments and of low relative density suggest the following four "rules" about morphological defects.

(i) Closed cellular structures with equiaxed cells having straight walls and borders with uniform thickness should exhibit stiffnesses and strengths approaching the limiting values expressed by (1) and (3) [20,23,29].

(ii) The cell size distribution is not a dominant factor.

(iii) Defects that degrade the elastic properties must normally be present with relatively high volume fraction.

(iv) Yielding initiates within small domains of spatially correlated defects.

3.2 Theoretical Results

(i) The distribution of material between the walls and the borders does not have an appreciable effect on the stiffness [20]. That is, upon thinning the walls (uniformly) and relocating the material at the nodes, the stiffness does not diminish until the walls become thin relative to the cell diameter. This insensitivity arises because bending effects are resisted by material placed at the nodes, thereby counteracting the reduction in membrane stiffness.

(ii) Cell wall curves and wiggles cause dramatic reductions in stiffness and yield strength [20,23]. It remains to quantify their role in a non-periodic structure. Calculations in 2D with non-periodic cells [34,35] have indicated that missing cell walls markedly diminish the yield strength. By inference, thin cell walls would have a similar effect.

3.3 Experimental Measurements

The deformations of cells have been monitored using two principal methods: (i) Surface deformations are followed by optical microscopy [22,31]. (ii) Internal cell deformations are reconstructed by using X-ray computed tomography (fig. 2)[22](CT-scan). In the latter, the deformation of defective cells fully-constrained by all of their neighbors can be systematically monitored; enabling the principal morphological defects to be catalogued. Strain mapping methods (fig. 3) vividly demonstrate that yielding is heterogeneous and occurs within bands about one cell diameter in width at stresses of order 1/3 the plateau strength. Moreover, these bands intensify and their number density increases as the stress elevates, until a peak is reached [31]. At the peak, plastic collapse occurs in one of the deformation bands. Each subsequent stress oscillation involves plastic collapse in successive bands.

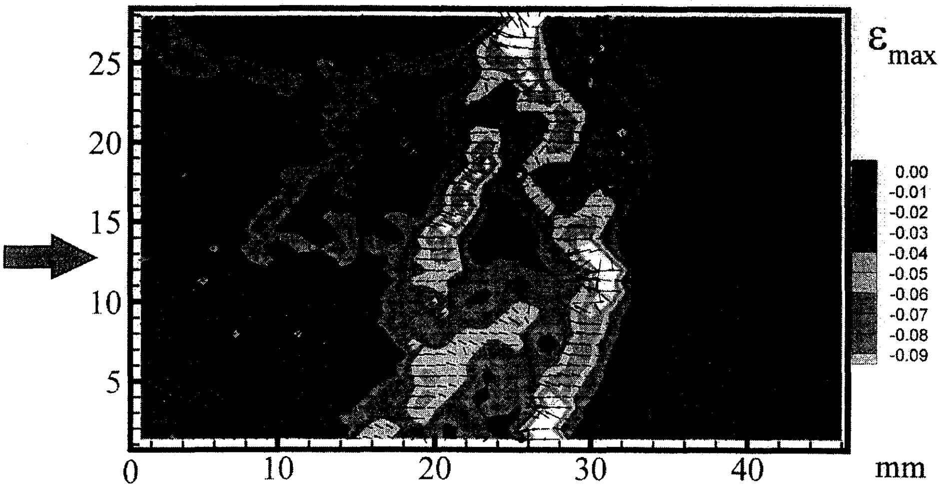

Fig. 2 Strain maps obtained for a closed cell Al material (Alporas) subject to uniaxial compression. Incremental strains $\Delta\varepsilon$ between the levels indicated on the stress/strain curve are presented. The maximum principal strain ε_{22} is shown and the vectors indicate the displacements. Note that virtually all of the strain occurs in the narrow deformation bands. Unloading is used to measure the modulus.

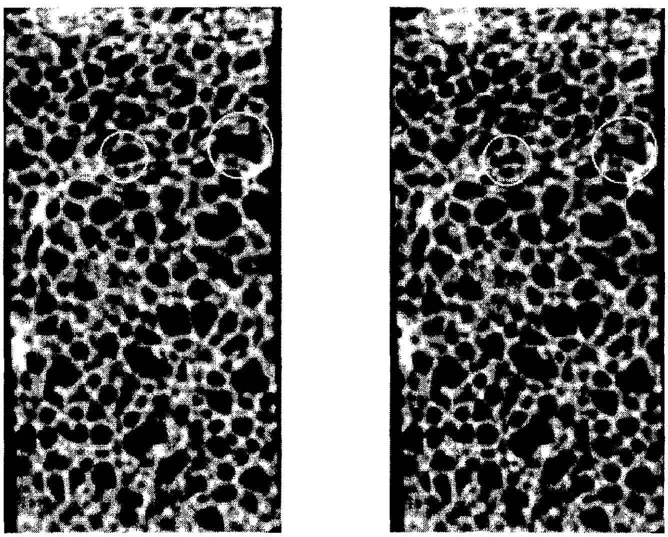

Fig.3 X-ray CAT scan images of cells before and after application of a compressive strain [22]. The circled regions identify those cells located within the deformation band that buckle upon straining. Note that the axial strain can be determined from the shortening of the top.

The X-ray results (fig. 3) have been instrumental in establishing two salient aspects of yielding within the deformation bands [22]. In accordance with "rules" (i) and (ii), equiaxed cells resist yielding, almost regardless of their size and cell-wall thickness distribution. The corollary is that large cells, if equiaxed, are not the source of the knock-down factor. Consistent with "rules" (i) and (iv), elliptical cells with their long axis normal to the loading direction are prevalent within deformation bands, regardless of size. Such cells, in cross-section, typically have T-shaped nodes with large entrained angles. Such nodes are subject to appreciable bending moments. The inference is that cell ellipticity results in bending effects that reduce the yield strength. A further, unsubstantiated inference is that elliptic cells, if present with sufficient spatial frequency (iii), would also diminish the stiffness.

4 Minimum weight structures

Table I. Structural Indices for Foam Core Systems

Indices	Column	Panel	Shell
Weight, ψ	$W/\Omega L^3$	$W/\Omega L^2 B$	$W/\Omega R^2 L$
Load (elastic), Π_e	$P/E_f L^2$	$P/E_f LB$	$P/E_f LR$
Load (plastic), Π_p	$P/\sigma_y L^2$	$P/\sigma_y LB$	$P/\sigma_y LB$

4.1 Structural Indices

Panels, shells and tubes subject to bending or compression have characteristics determined by structural indices [10,11,24,25,39-42]. These are obtained by deriving expressions for the stresses, displacements and weights in terms of the loads, dimensions, elastic properties and core densities. The details depend on the configuration, the loading and the potential failure modes, as elaborated below. There are a relatively small number of indices, based on weight, ψ, and load, Π. These can be expressed either in non-dimensional form (table I) or in convenient dimensional forms. For bending, it is convenient to define an additional structural index: the stiffness index, S, related to the elastic load index, Π_e by:

$$S = \Pi_e (L/\delta) = (P/\delta) B / E_s \tag{5}$$

where P is the load, δ the deflection, L the span and E_s the Young's modulus for the material comprising the cellular medium.

When optimizations are conducted simultaneously for weight and core density, explicit weight and deflection ratios result which, thereafter, greatly simplify determination of the relationships between the structural indices [1,42]. For example, stiffness-limited, laterally-loaded panels containing a core with stiffness characterized by (5) exhibit minimum overall weight when the weight of the face sheets is 1/4 that for the core. At this minimum, the contribution to the deflection by core shear is exactly twice that contributed by stretching the face sheet.

4.2 Stiffness-Limited Sandwich Structures

Panels that experience lateral loads are often stiffness limited. Stiffness also affects the natural vibration frequencies. That is, high stiffness at low weight increases the resonant frequencies, thereby facilitating their avoidance. Choosing minimum weight configurations is relatively straightforward whenever the design loads allow choices entirely within the elastic range. The basic concepts can be found in several literature sources. The key results are reiterated to establish the procedures, as well as to capture the most useful results. For all bending problems, a series of non-dimensional coefficients, designated A_i, relate the deflections to the moments. These have been comprehensively summarized elsewhere [1,2,46]. A more complete optimization is possible if the density of the core is treated as a free variable, because the displacements contributed by core shear are twice those from bending and the core weight is 4 times that of the face sheets. These ratios give the relationship:

$$S = (A_1/60)(c/L)^2 \ \Psi \tag{6}$$

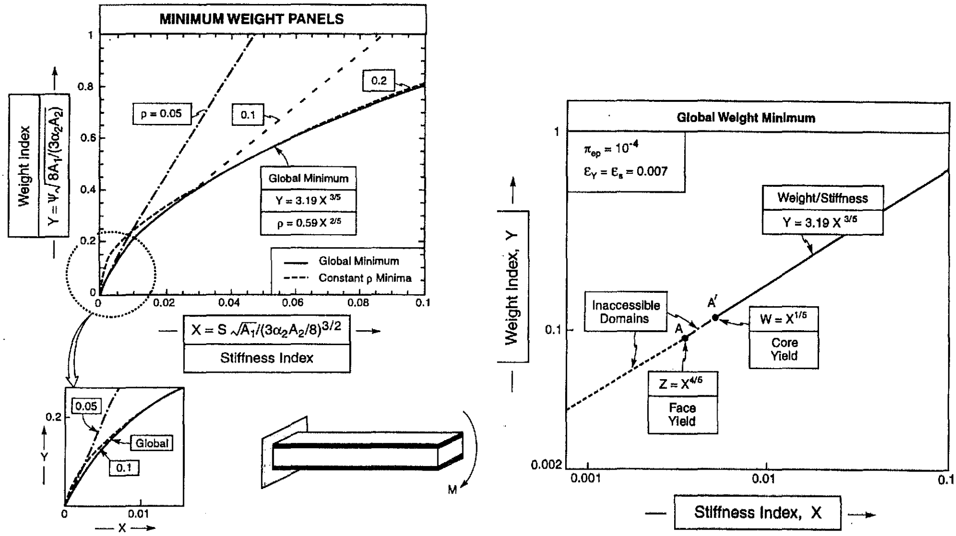

Fig.4: Minimum weight analysis for stiffness-limited, laterally loaded panels. (a) A cross plot of the minimum weight and stiffness indices showing the global minimum, as well as minima for three fixed densities ($\rho = 0.05$, 0.1 and 0.2). Note that, for the global minimum, the core density is given by $\rho = 0.59X^{2/5}$. (b) A schematic illustrating the domains wherein yielding of either the face sheets or the core prohibit use of elastic analysis. For the choice of load index $(\Pi_p = 10^{-4})$ and yield strain $(\varepsilon_Y = \varepsilon_s = 0.007)$, core yielding will arise in minimum weight panels designed at stiffness below A'. The corresponding point for face yielding is A.

Here it is assumed that the core and face sheets are made from the same alloy. At the weight minimum, the core thickness is explicitly related to the stiffness by:

$$\frac{c}{L} = 2\left[\frac{18\alpha_2 A_2 S}{A_1^2}\right]^{1/5} \tag{7}$$

Substituting c/L into (6) gives the inter-relationship:

$$\Psi = \frac{15S^{3/5}}{A_1^{1/5}(18\alpha_2 A_2)^{2/5}} \tag{8}$$

For plotting purposes, it is convenient to re-express (8) in the form (fig. 4a),

$$Y = 3.19X^{3/5} \tag{9}$$

where $Y = \Psi\sqrt{8A_1/3\alpha_2 A_2}$ and $X = S\sqrt{A_1}/(3\alpha_2 A_2/8)^{3/2}$. For each stiffness, there is a corresponding relative density for the core:

$$\rho = 0.59X^{2/5} \tag{10}$$

There is also an explicit face thickness,

$$\frac{d_f}{L} = \frac{A_1}{96\alpha_2 A_2}\left(\frac{c}{L}\right) \tag{11}$$

Application of these weight diagrams is limited by the occurrence of yielding, either of the face sheets or in the core, and by face wrinkling [1,11]. For these reasons, configurations having lower stiffness than A or A' on fig. 4 cannot be realized at the weights given by (8).

Structures that realize global weight minima are those requiring *high stiffness.* At lower stiffnesses, because of the thinner face sheets (11) and lower core densities (10), yielding is likely to intervene. For yielding to be avoided, the weight must be increased above the minimum by increasing either the face sheet thickness or the core density.

Competition for sandwich systems is comprised principally of waffle-stiffened panels. Comparison with the optimized sandwich panel yields, *at equivalent weight:*

$$\frac{d_s}{c} = \frac{\sqrt{6}}{5} \tag{12}$$

where d_s is the depth. This result is *stiffness independent.* Accordingly, a waffle panel made from the same material as a sandwich panel has a *slightly smaller* overall thickness, at the same weight and stiffness. The choice, therefore, depends entirely on manufacturing cost and durability.

4.4 Cylindrical Shells

Strength-limited sandwich structures can be weight competitive with stiffener reinforced designs: the lowest weight designs in current usage [43-45]. Shells are a more likely candidate for sandwich construction than axially compressed panels or columns. This preference arises because both hoop and axial stresses are involved, enabling the isotropy of sandwich panels to be exploited. There are two basic requirements for sandwich shells. (i) Sufficient core shear stiffness is needed for adequate buckling strength. (ii) The shear yield strength of the metal foam must be large enough to maintain the buckling resistance of the shell, particularly in the presence of imperfections. Numerical methods are needed to determine minimum weights of both sandwich and reinforced systems. *Simple analytical estimates are not possible.* Some prototypical results are presented to illustrate the configurations wherein sandwich construction may be preferred.

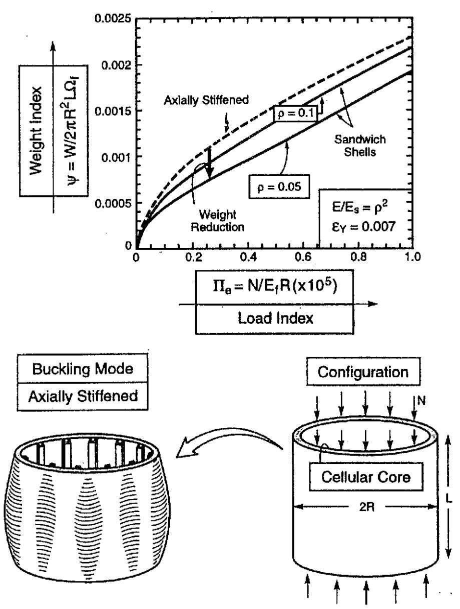

Fig.5 Minimum weight comparisons for strength-limited, axially compressed cylindrical sandwich shells at fixed density (with $\alpha_2 = 1$) having dimension $L/R_s = 1$, compared with those for a shell with inside stiffeners.

One configuration comprises cylindrical shells subject to axial loads (fig. 5), optimized with respect to d_f and c, subject to prescribed core density. They regard the fully dense core material as identical to the face sheet material and use a core with stiffness at the low end of the range found for commercial materials. The face sheets are elastic-perfectly plastic with compressive yield strength σ_Y. Note that, at the optimum weight, and in the range where the face sheets experience yield, the compressive stress in the face sheets associated with elastic buckling is coincident with the yield strength in compression, σ_Y. The weight index has been determined at a representative yield strain for Al alloys. These results are independent of the length of the cylinder. The buckling mode is axisymmetric.For comparison the structural performance has been calculated for an optimally-designed, axially-stiffened cylindrical shell with hat-shaped stiffeners located on the inside. These results apply to a shell segment located between rings spaced a distance L apart, with $L/R_s = 1$. A lower L/R_s would have a lower weight index, and vice versa. Note that, over the range plotted, the shell buckles elastically for the chosen yield strain. (Shells with stiffeners on the outside of the cylinder have somewhat greater buckling strength and, thus, a lower weight index. But, outside stiffening is often excluded for other reasons.) *This example illustrates that metal foam core sandwich shells can have a competitive advantage over established structural methods of stiffening, particularly at relatively low structural indices.*

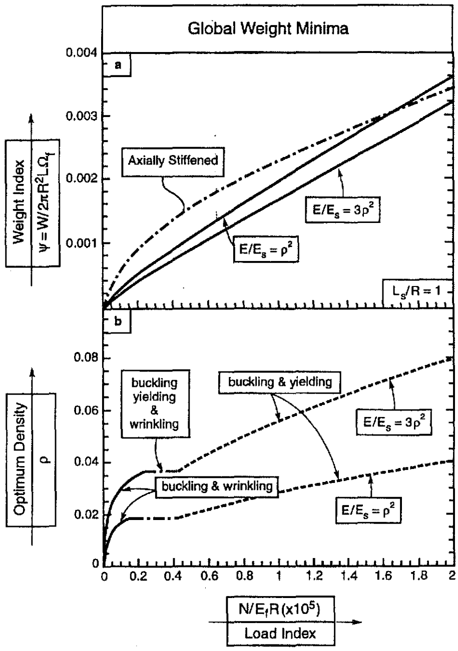

Fig.6 (a) Minimum weights of sandwich construction for two core stiffness behaviors compared with those for stiffened shells. Here the minimization has been conducted on both weight and core density. The stiffnesses were chosen to encompass those found experimentally ($1 \lessapprox \alpha_2 \lessapprox 4$, fig.1). (b) The relative densities at the weight minimum represented by (a).

To pursue the subject further, the sandwich shells have been optimized with respect to relative core density ρ, as well as d_f and c. Simultaneously, the consequence of using a core with superior stiffness is addressed by assuming a core having properties comparable to the best commercial materials. The results for the fully optimized foam core sandwich shells are plotted in fig. 6 with accompanying plots for the optimal relative density of the core. For reference, the result for the optimally designed cylindrical shell with axial hat-stiffeners is repeated from fig. 5. This comparison illustrates both the weight superiority of foam core sandwich shells over conventional shell construction and the potential benefit to be gained by using a core material with the best available stiffness. At the lowest core densities, face wrinkling is expected to become the weight limiting failure mechanism. While analysis of this mechanism would be needed to establish specific weight benefits, optimized sandwich construction is still expected to afford lower weights than hat-stiffening.

4.5 Synopsis

(i) Determine the constraints that govern the structure and, in particular, whether it is stiffness or strength controlled.

(ii) If stiffness is the dominant, there is a relatively straightforward procedure for determining minimum weights. This entails using the formulae summarized in the tables. It is important to realize that there will *always be lighter configurations* (especially optimized honeycomb or waffle panels). Those configurations should be explicitly identified, whereupon a manufacturing cost and durability comparison can be made that determines the viability of sandwich construction. Other qualities of the cellular metal may bias the choice. Moreover, it is important to calculate the domains wherein the weights based on elasticity considerations cannot be realized, because of the incidence of "inelastic" modes: face yielding, core yielding, face wrinkling.

(iii) When strength (particularly buckling) controls the design, the rules governing sandwich construction are less well formulated. In general, numerical methods are needed to compare and contrast this type of construction with stiffened systems. Some general guidelines are given in the Manual [2]; these facilitate deliberations about loadings and configurations most likely to benefit from sandwich construction. Configurations unlikely to benefit are also described. It is recommended, that, wherein benefits seem likely, detailed simulations and testing should be used to assess the viability of sandwich construction.

5 Heat transfer media

The thermal conductivity of cellular metals is quite large compared with their polymer and ceramic counterparts [1]. Moreover, their thermal diffusivity is comparable to that for dense metals [2,16]. These materials are thus of little interest for thermal insulation. Instead, advantage can be taken of their high thermal diffusivity for use as heat exchange media [13,14]. In order to calibrate the heat transfer capabilities of cellular metals and, accordingly, extrapolate into new domains, a basic model is needed. There are several options. One consistent with measurements, regards the cellular medium as a variant on a bank of cylinders. That is, the spatial variations in the temperatures of the solid and fluid have similar forms for the cellular metal medium and the cylinders. But, the coefficients differ. Accordingly, proportionality constants are needed to represent the effective thermal conductivity and the effective local heat transfer coefficient.

A trend toward higher heat dissipation is established, as either the ligament diameter d becomes smaller or the relative density ρ increases. This trend reflects the higher internal surface area as d decreases and the greater heat conduction cross-section as ρ increases. However, this effect is countermanded by an increased drop in the pressure needed to force the fluid through the medium. This pressure drop tends to increase as the surface area to volume ratio increases. Accordingly, there is an optimum cell structure that depends explicitly on the application and its specifications.

For any system there is a trade-off between heat flux and pressure drop. A cross plot of these two quantities in accordance with the non-dimensional parameters defined by the model (table II) illustrates this (fig. 7). The solid lines indicate the trend between heat dissipation and pressure drop as the relative density increases, at fixed cell size. The dotted lines indicate the effect of cell size at fixed density. Each intersection point represents a specific cellular material.

Table II. Non-Dimensional Parameters Governing the Performance of Cellular Metal Heat Dissipation Media

Heat Flux	$\tilde{Q} = Q / k[T_1 - T_o]$
Prandtl Number	$\Pr = v_f / a_f$
Reynolds Number	$\tilde{\mathrm{Re}} = vL / v_f$
Cell Wall Thickness	$\tilde{d} = d / L$
Foam Thickness	$\tilde{D} = D / L$
Nusselt Number	$Nu = Bik / k_f$
Thermal Conduction	$\tilde{K}_f = \sqrt{k_f / k}$
Power Dissipation	$\tilde{p} = \Delta p v D L^2 / \rho_f v_f^3$

The preferred material domain is indicated on fig. 7. It is preferred because such materials achieve high heat dissipation at moderate pressure drop: that is, for cell size in the meso-range and with relative densities of order, $\rho \approx 0.2$. The particular material of choice can only be found by coupling fig. 7 with the operating characteristics (back pressure and flow rate) of fans/blowers used to circulate the fluid. Such analysis has revealed that the trade-off between pressure drop and heat dissipation achievable with cellular metals is more propitious than that for conventional fin-pin arrays [2], enabling the design of substantially more compact heat sinks.

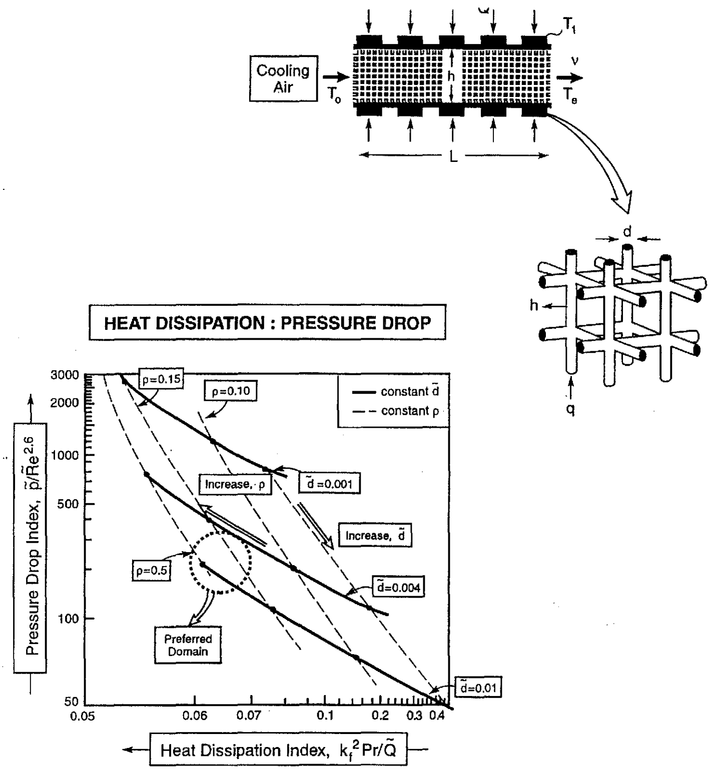

Fig. 7 A schematic of an open cell metal used as a heat dissipation medium, e.g., for cooling high power electronics. Also shown is the trade-off between pressure drop and heat flux, with the preferred material domain indicated.

6 Summary

The connections between the morphological quality of cellular metals and the requirements for their implementation comprise: (i) those insensitive to the thermomechanical properties of the material and (ii) others that are strongly influenced by cellular material quality. This distinction partitions the connection between manufacturing and implementation.

(a) Several applications categories are relatively insensitive to morphological quality, provided that some reasonable minimum is consistently achieved. These comprise energy absorption applications and some ultralight panels and tubes. The latter category includes some stiffness-limited structures, as well as strength limited configurations subject to low imperfection sensitivity. (b) Other applications categories require that the cellular material have the best achievable thermomechanical properties. One category comprises imperfection sensitive ultralight shells and circular tubes that operate in the elastic range. In such cases relatively high strength cores, approaching the best achievable, are essential to the realization of substantial weight savings. Another category comprises open cell heat dissipation media.

Within these overall material property benchmarks, comparisons with competing materials and systems suggest the following implementation opportunities.

(i) For heat dissipation purposes, cellular metals are unique. Moreover, there are substantial opportunities to greatly improve their thermal performance by tailoring cell size and density. The manufacturing challenge is demanding, but justified by the performance benefit.

(ii) Strength and stiffness limited ultralight structures designed within the elastic range all exhibit a domain wherein weight benefits arise from the use of thin sandwich construction comprising cellular metal cores. A subset has sufficient performance benefit to justify implementation. Others may have utility because of lower manufacturing cost. Preliminary attempts at defining structures that provide weight savings have identified panels and shells as opportunities. The greatest benefits appear to arise with relatively long strength-limited shells subject to axial compression. There also appear to be opportunities for stiffness limited panels that experience lateral loads. There are no benefits for compression structures designed with a load index in the plastic range. The requirements on the mechanical properties of the cellular material are themselves subject to the imperfection sensitivity of the structure. For imperfection insensitive structures, the dictates on properties are minimal. But, the benefits from using a cellular core are also small. Conversely, imperfection sensitive structures, such as cylindrical shells, benefit most from having cellular cores with properties approaching the best achievable levels, with no knockdown. Cellular metal sandwich construction would provide even greater weight benefit if the density of the core could be substantially decreased below presently available materials ($\rho \ll 0.1$), subject to mechanical properties that approach best-achievable levels. Attainment of such materials constitutes a longer range manufacturing objective.

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